**1st and 2nd Derivatives**

The first derivative *f’* of *f* tells us where *f* is increasing and where *f* is decreasing, while the second derivative *f”* of *f* tells us where *f* is concave upward and where *f* is concave downward.



**Finding Relative Extrema (1st Derivative Test)**

* Determine the critical points of *f*
	+ Find the 1st derivative of *f*
	+ Determine where *f’(x)=0* or doesn’t exist (kink or vertical tangent)
* Determine the sign of *f’(x)* in each interval generated by the critical points
	+ *f’(x)* changes sign from + to -, relative maximum
	+ *f’(x)* changes sign from – to +, relative minimum
	+ *f’(x)* doesn’t change sign, no relative extremum

**Concavity of a Function**

* *f* is concave up on (a,b) if *f’* is increasing on (a,b)
* *f* is concave down on (a,b) if *f’* is decreasing on (a,b)
	+ f”(x)>0for each value of x in (a,b), *f* is concave up on (a,b)
	+ f”(x)<0for each value of x in (a,b), *f* is concave down on (a,b)

**Steps to determine concavity**

1.      Determine the values of *x* for which *f”* is 0 or undefined

2.      Use these critical values to determine the sign of each interval for *f”*

**Inflection Points**

A critical value for *f”* is an inflection point if the concavity

changes at this value.

**Steps to find inflection points**

1. Find the 2nd derivative
2. Use the 2nd derivative to test for concavity (see above)
3. If there is a change in sign from left to right of the test value, it’s an inflection point.

**Second Derivative Test**

The second derivative test is an alternative procedure for finding whether a function f has a relative extremum at a critical value *c*. It is applicable only when *f”* exists and is therefore less versatile than the first derivative test. Also, it is inconclusive when *f”(c)* is equal to zero at a critical point of *f* and is inconvenient to use when *f”* is difficult to compute.

However, when *f”* is easy to compute, use the second derivative test as this involves just the evaluation of *f”* at the critical point(s) of *f*.

1. Determine *f’(x)* and *f”(x).*

2. Find all the critical points of *f* at which *f’(x)=0.*

3. Determine *f”(c)* for each critical point *c*.

a. If *f”(c)<0*, then *f* has a relative maximum at *c*.

b. If *f”(c)>0*, then *f* has a relative minimum at *c*.

c. If *f”(c)=0*, the test fails; that is, it is inconclusive.

